Unit III

Hypothesis Testing

1. Introduction to Hypothesis

In statistics, a **hypothesis** is a specific statement or assumption about a population parameter (such as mean, proportion, or variance) that can be tested using data obtained from a sample.

Hypothesis testing helps researchers or decision-makers draw conclusions about populations based on sample information. It provides a **scientific and objective method** for decision-making under uncertainty.

Example:
A company claims that the average lifetime of its LED bulbs is **10,000 hours**.
To verify this claim, a sample of 50 bulbs is tested. Based on the sample mean and standard deviation, the researcher decides whether to accept or reject the company's claim.

Hence, hypothesis testing helps in:

- Evaluating business claims (e.g., product quality, sales performance, service time).
- Comparing two populations (e.g., male vs. female income levels).
- Assessing the effect of a new policy or training program.

2. Types of Hypotheses

(a) Null Hypothesis (H_0)

- Represents a statement of **no effect**, **no difference**, or **status quo**.
- It assumes that any observed difference is due to **chance or random error**.
- The null hypothesis is **assumed true** until statistical evidence suggests otherwise.

Example:
A soft drink company claims that the mean fill of its bottles is **300 ml**.
→ H_0: $\mu = 300$ ml
(This means the average quantity filled is 300 ml.)

(b) Alternative Hypothesis (H_1 or Ha)

- Represents a **contradiction** to the null hypothesis.
- It indicates that a real difference or relationship exists.
- The researcher's goal is to **gather enough evidence to support H_1**.

Example:
Continuing the above case, if we believe the mean fill is different from 300 ml:
→ H_1: $\mu \neq 300$ ml

If we suspect bottles are underfilled:
→ H_1: $\mu < 300$ ml

If we suspect bottles are overfilled:
→ H_1: $\mu > 300$ ml

3. Steps Involved in Hypothesis Testing

I. State the Hypotheses

The first step in hypothesis testing is to clearly state the null and alternative hypotheses. The **null hypothesis (H_0)** assumes that there is **no significant difference or relationship** between the variables under study, serving as the default assumption.

The **alternative hypothesis (H_1 or Ha)** proposes that a significant difference or relationship **does exist**.

For example, in testing a new marketing strategy, H_0 may state that the strategy has no effect on sales, while H_1 states that it increases sales. Stating hypotheses provides a clear direction for the statistical test.

II. Select the Level of Significance (α)

The level of significance represents the **probability of making a Type I error**, i.e., rejecting a true null hypothesis. It defines the confidence level of the test. Commonly used significance levels are **0.01, 0.05, and 0.10**, depending on how strict the test needs to be.

For example, an $\alpha = 0.05$ means there is a 5% risk of rejecting H_0 when it is actually true. The smaller the value of α, the more conservative the test, meaning stronger evidence is required to reject the null hypothesis.

III. Select the Appropriate Test Statistic

Once the hypotheses and significance level are set, the next step is to choose the appropriate **statistical test** based on the data type, sample size, and population characteristics. Common tests include the **Z-test** (for large samples with known population variance), **t-test** (for small samples), **Chi-square test** (for categorical data and independence), and **F-test** (for comparing variances or in ANOVA). The correct selection ensures that the test results are statistically valid and suitable for the research question.

IV. Calculate the Test Statistic

In this step, the chosen test statistic is computed using the sample data and the relevant formula. The test statistic (such as Z, t, χ^2, or F) measures the degree of difference between the observed sample result and the hypothesized population value. The formula used depends on the type of test and data available.

For example, in a Z-test for means, the statistic is calculated as $Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$. This calculated value is then used for comparison with the critical value.

V. Determine the Critical Value

The critical value is obtained from statistical distribution tables corresponding to the **chosen α level** and **degrees of freedom**. It defines the **threshold** beyond which the null hypothesis will be rejected.

For example, for a two-tailed Z-test at $\alpha = 0.05$, the critical values are ± 1.96. These values divide the sampling distribution into acceptance and rejection regions. Determining the correct critical value is crucial because it sets the benchmark for decision-making in the hypothesis test.

VI. Make a Decision

After computing the test statistic and identifying the critical value, the next step is to compare the two. If the **calculated value exceeds the critical value**, the null hypothesis (H_0) is **rejected** in favor of the alternative hypothesis (H_1).

If it does not exceed the critical value, we **fail to reject H_0**. This decision step determines whether the observed results are statistically significant or simply due to random variation. It is the key step that leads to an evidence-based conclusion.

VII. Draw Conclusion

The final step involves interpreting the statistical decision in the context of the research problem. If H_0 is rejected, it means there is **sufficient evidence** to support the alternative hypothesis. If H_0 is not rejected, it implies that there is **not enough evidence** to conclude a significant difference or effect. The conclusion should be written in simple language, relating it to the original problem. For example, "The analysis shows that the new training program significantly improves employee productivity at a 5% level of significance."

4. Type I and Type II Errors

When we conduct **hypothesis testing**, we make decisions about population parameters based on sample data. However, these decisions may not always be correct because of **sampling variability**, which can lead to **two types of errors**.

Decision	H_0 True	H_0 False
Reject H_0	Type I Error (α)	Correct Decision ($1 - \beta$)
Do Not Reject H_0	Correct Decision ($1 - \alpha$)	Type II Error (β)

(a) Type I Error (α)

- **Meaning:** Occurs when we *reject a true null hypothesis (H_0)*.
- It means we are concluding that an effect or difference exists **when in fact it does not**.
- The **probability of committing a Type I error** is called the **Level of Significance (α)**, usually set at 0.05 or 0.01.

Example:

Suppose a manufacturer claims that the average lifetime of their LED bulbs is **10,000 hours**.

- **Null hypothesis (H_0):** $\mu = 10{,}000$ hours
- **Alternative hypothesis (H_1):** $\mu \neq 10{,}000$ hours

If the researcher rejects H_0 based on the sample data — even though the true mean lifetime really is 10,000 hours — then a **Type I Error** has occurred.

Interpretation: We wrongly conclude that the bulbs do not last 10,000 hours when, in fact, they do.

(b) Type II Error (β)

- **Meaning:** Occurs when we *fail to reject a false null hypothesis (H_0)*.
- It means we are **missing a real effect or difference** that truly exists.
- The **probability of committing a Type II error** is represented by **β**.

Example:

Continuing the same LED bulb case:

- **Null hypothesis (H_0):** $\mu = 10{,}000$ hours
- **Alternative hypothesis (H_1):** $\mu \neq 10{,}000$ hours

If the true mean lifetime of bulbs is actually **9,000 hours**, but we fail to reject H_0 (we accept that $\mu = 10{,}000$), then a **Type II Error** occurs.

Interpretation: We mistakenly believe the bulbs last 10,000 hours when in reality, their average life is only 9,000 hours.

5. Level of Significance (α)

- The level of significance (α) represents the **maximum probability of making a Type I error** that a researcher is willing to accept.
- Common values: **0.05 (5%)** or **0.01 (1%)**.
- A smaller α implies **stronger evidence** is required to reject H_0.

Example:
At 5% significance level, we are willing to accept a 5% chance of wrongly rejecting H_0.

6. One-Tailed and Two-Tailed Tests

The choice of one-tailed or two-tailed test depends on the **direction** of the research hypothesis.

(a) One-Tailed Test

Used when the alternative hypothesis specifies a **direction** (greater than or less than).

Type	**Form of H_1**	**Critical Region Location**	**Example**
Right-tailed	$H_1: \mu > \mu_0$	Right side of distribution	Testing if new process increases output
Left-tailed	$H_1: \mu < \mu_0$	Left side of distribution	Testing if average weight has decreased

Illustration:
A company claims the average life of batteries is 100 hours. A researcher suspects it is less.
→ $H_0: \mu = 100$
→ $H_1: \mu < 100$ (Left-tailed test)

(b) Two-Tailed Test

Used when the alternative hypothesis **does not specify a direction**; it only states that there is a difference.

Type	**Form of H_1**	**Critical Region Location**	**Example**
Two-tailed	$H_1: \mu \neq \mu_0$	Both sides of the curve	Testing if mean income differs from ₹40,000 (could be higher or lower)

Illustration:
A bank claims the average waiting time is 10 minutes. The researcher believes it might be either higher or lower.
→ $H_0: \mu = 10$
→ $H_1: \mu \neq 10$ (Two-tailed test)

7. Example of Hypothesis Testing

Example:
A sample of 36 employees shows an average monthly salary of ₹42,500 with a standard deviation of ₹3,600.
The management claims that the average salary is ₹40,000.
Test at 5% significance level whether the claim is valid.

Solution:

1. H_0: $\mu = 40{,}000$
   H_1: $\mu \neq 40{,}000$ (two-tailed test)
2. Given:
   Sample mean ($\bar{x}$) = 42,500
   Population mean (μ) = 40,000
   $s = 3{,}600$, $n = 36$, $\alpha = 0.05$
3. Test statistic:

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{42500 - 40000}{3600/6} = \frac{2500}{600} = 4.17$$

4. Critical value ($z_{a/2}$) = ±1.96

Since **4.17 > 1.96**, we **reject H_0**.

Conclusion:
There is sufficient evidence to conclude that the average salary is significantly different from ₹40,000.

8. Summary Table

Concept	**Symbol / Definition**	**Remarks**
Null Hypothesis	H_0	No difference or effect
Alternative Hypothesis	H_1 / Ha	There is a difference
Type I Error	α	Rejecting true H_0
Type II Error	β	Failing to reject false H_0
Significance Level	α	Common: 0.05 or 0.01
One-Tailed Test	Directional	H_1: $\mu > \mu_0$ or $\mu < \mu_0$

Two-Tailed Test	Non-directional	$H_1: \mu \neq \mu_0$

9. Importance of Hypothesis Testing in Business

- Helps **verify company claims** (e.g., production output, defect rate).
- Supports **decision-making** under uncertainty.
- Assists in **quality control and performance measurement**.
- Provides **quantitative evidence** for managerial and policy decisions.
- Widely used in **market research, finance, and economics** to validate assumptions.