HYPOTHESIS TESTING – ADVANCED APPLICATIONS

1. Testing Hypothesis about the Difference Between Two Means – Z-Test

When comparing **two population means** (say, from two factories, batches, or time periods), we use the **Z-test** if:

- Both sample sizes are **large ($n \geq 30$)**
- Population variances (σ_1^2, σ_2^2) are **known** or **approximately equal**

Formula

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

If under $H_0: \mu_1 = \mu_2$,

$$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Example 1

Two brands of bulbs, A and B, are tested for average life.

Brand	Sample Size	Mean Life (hrs)	SD (σ)
A	60	1200	100
B	50	1230	120

Test at 5% significance whether the mean lives differ.

Solution:

$$H_0: \mu_1 = \mu_2, H_1: \mu_1 \neq \mu_2$$

$$Z = \frac{1200 - 1230}{\sqrt{100^2/60 + 120^2/50}} = \frac{-30}{\sqrt{166.7 + 288}} = \frac{-30}{21.4} = -1.40$$

At 5% level, critical $Z = \pm 1.96$
Since $|-1.40| < 1.96$ → **Fail to reject H_0.**

Conclusion: There is no significant difference in the mean life of the two brands.

Example 2

The mean salary of 45 MBA graduates from University X is ₹56,000 (σ_1=₹4,000), while from University Y (n_2=40) is ₹58,000 (σ_2=₹4,500).
Test if their average salaries differ at 5%.

$$Z = \frac{56000 - 58000}{\sqrt{4000^2/45 + 4500^2/40}} = \frac{-2000}{\sqrt{355555 + 506250}} = \frac{-2000}{970.8} = -2.06$$

$|Z| = 2.06 > 1.96 \Rightarrow$ **Reject H_0.**

Conclusion: The mean salaries of the two universities differ significantly.

2. Testing Hypothesis about a Population Proportion

Used when the population proportion (e.g., defective items, successes, voters) is to be verified.

Formula

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

Where

- $\hat{p} = \frac{x}{n}$(sample proportion)
- p= hypothesized population proportion

Example 1

A company claims that 90% of its products are defect-free.
In a random sample of 100 products, 85 were defect-free. Test the claim at 5% significance.

$$p = 0.9, \hat{p} = 0.85, n = 100$$
$$Z = \frac{0.85 - 0.9}{\sqrt{0.9(0.1)/100}} = \frac{-0.05}{0.03} = -1.67$$

At 5% level, critical Z = ±1.96 → |Z| < 1.96
Conclusion: The claim is **not rejected**; data supports the company's statement.

Example 2

In a survey, 40 out of 100 respondents said they prefer online shopping. Is it justified to conclude that less than 50% prefer it? (α = 0.05)

$$p = 0.5, \hat{p} = 0.4, n = 100$$

$$Z = \frac{0.4 - 0.5}{\sqrt{0.5(0.5)/100}} = \frac{-0.1}{0.05} = -2.0$$

For one-tailed test, Z(critical) = −1.645
Since −2.0 < −1.645 → **Reject H_0**

Conclusion: Less than half of respondents prefer online shopping.

3. Testing Hypothesis about the Difference Between Two Proportions

Used to check if **two groups differ in proportion** (e.g., men vs. women preference).

Formula

$$Z = \frac{p_1 - p_2}{\sqrt{p(1-p)(\frac{1}{n_1} + \frac{1}{n_2})}}$$

Where

$$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

Example 1

In a survey, 60 of 100 men and 72 of 120 women prefer Brand A.
Test if preference differs significantly.

$$p_1 = 0.6, p_2 = 0.6, p = \frac{100(0.6) + 120(0.6)}{220} = 0.6$$

$$Z = \frac{0.6 - 0.6}{\sqrt{0.6(0.4)(1/100 + 1/120)}} = 0$$

Conclusion: No difference in preference between men and women.

Example 2

60 out of 100 urban consumers and 50 out of 100 rural consumers like a product. Test if preference differs.

$$p_1 = 0.6, p_2 = 0.5, p = 0.55$$

$$Z = \frac{0.6 - 0.5}{\sqrt{0.55(0.45)(1/100 + 1/100)}} = \frac{0.1}{0.0707} = 1.41$$

|Z| < 1.96 → **Fail to reject H_0**
Conclusion: No significant difference between urban and rural consumers.

4. Testing Hypothesis about Two Population Variances – F-Test

Used to test if two samples come from populations with equal variances.

Formula

$$F = \frac{S_1^2}{S_2^2} \text{ where } S_1^2 > S_2^2$$

Example 1

Sample 1 (n_1=10): variance = 20
Sample 2 (n_2=8): variance = 10
Test equality of variances at 5% level.

$$F = \frac{20}{10} = 2.0$$

Critical F(0.05, 9,7) ≈ 3.79
Since 2.0 < 3.79 → **Fail to reject H_0**

Conclusion: Variances are equal.

Example 2

Two samples give:
$S_1^2 = 36$, $n_1 = 15$; $S_2^2 = 12$, $n_2 = 12$

$$F = \frac{36}{12} = 3.0$$

F(0.05, 14,11) = 2.82
Since 3.0 > 2.82 → **Reject H_0**

Conclusion: Variances differ significantly.

5. Small Sample Theory – t-Test

Used when **sample size < 30** and population σ is **unknown**.
It relies on the **Student's t-distribution**.

(a) One-Sample t-Test

$$t = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

Example 1

A manufacturer claims that the average weight of a product is 50g.
A sample of 9 items has mean 48g and SD 4g.
Test at 5% significance.

$$t = \frac{48 - 50}{4/\sqrt{9}} = \frac{-2}{1.33} = -1.50$$

$t(0.05, 8) = \pm 2.306$
Since $|t| < 2.306$ → **Fail to reject H_0**

Conclusion: Mean weight does not differ significantly from 50g.

Example 2

Mean life of 10 bulbs = 1500 hrs, SD = 120 hrs. Is this different from the claimed 1600 hrs?

$$t = \frac{1500 - 1600}{120/\sqrt{10}} = -2.63$$

$t(0.05, 9) = \pm 2.262$ → $|t| >$ t-critical
Conclusion: Mean life differs significantly from 1600 hrs.

Comparison Summary

Test	Used For	Sample Size	Distribution	Key Use
Z-test	Mean/Proportion (large sample)	≥30	Normal	Known σ
t-test	Mean (small sample)	<30	t-distribution	Unknown σ
F-test	Variance comparison	Any	F-distribution	Compare variability
Z (Proportion)	Success ratios	≥30	Normal	Proportion data