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1. Learning Outcomes

After studying this module, you shall be able to

- Understand the concept of Arbitrage Pricing Theory
- Explain the Principle of Arbitrage
- Learn the arbitrage mechanism
- Know the arbitrage portfolios
- Compare APT and CAPM

2. Introduction

The presence of price discrepancies among equivalent securities in different markets can be used to earn riskless profits through simultaneous purchase and sale of the securities in these markets. Such an exploitation of the security mispricing to earn profits is called arbitrage. The equilibrium capital markets hold the basic principle of 'no arbitrage condition' i.e. the equilibrium market prices do not allow for the arbitrage opportunities. Perhaps, there would be pressure to restore equilibrium in case the actual security price allow for arbitrage. Possibly a simple relationship between expected return and risk could be derived by combining the factor model and the no arbitrage condition of the equilibrium capital market. Such a trade - off between risk and return is known as Arbitrage Pricing Theory (APT).

3. Arbitrage Pricing Theory

The CAPM is an excellent representation of the process by which stock prices are determined, but it is not perfect. The underlying assumptions of the CAPM may not hold well in the real world. Also, there are serious doubts about its testability. Moreover, CAPM is a single period model based on the market factor influencing the security's returns. An alternative to the above equilibrium asset pricing model is the multifactor asset pricing model purely based on the arbitrage arguments. It is the Arbitrage Pricing Theory (APT) model developed by economist Stephen Ross in 1976.

The key point behind APT is a logical statement that security returns are not based on one single factor of the market, rather it is influenced by multiple macro-economic factors where sensitivities to each factor is represented by a factor specific beta coefficient. The APT model proposes that there exists a linear relationship between the return on asset and the number of risk factors. Although, the APT model does not specify what these risk factors are, but asset returns are linearly related to the risk factors.

In an article in the *Financial Analysis Journal, Richard Roll and Stephen Ross* state specifically that APT assumes that a security's long run return is "directly related to its sensitivities to unanticipated changes in the four macroeconomic variables – (i) inflation, (ii) industrial production, (iii) risk premium, and (iv) the scope of the term structure of interest rates. Assets, even if they have the same CAPM beta, will have different patterns of sensitivities to these systematic factors."

Richard Roll and Stephen Ross "The Arbitrage Pricing theory Approach to Strategic Portfolio Planning", *Financial Analysis Journal,* May, June 1984. 14-26

APT also emphasizes the ***Law of One Price*** which implies that the two securities will command the market price given that they are equivalent in all economically relevant respects. In other words, in an equilibrium market (with no arbitrage condition), the two identical securities having same degree of risk will have same price i.e. will have same return in the market in the long run. However, in the short run disequilibrium, there may be differences in the market price of securities having same amount of risk. This would induce arbitrage activities by the arbitrageurs, until the arbitrage opportunities are eliminated.

3.1 Assumptions of APT Model

The arbitrage pricing theory relies on three assumptions:

i. A factor model can be used to describe the relation between risk and return of a security,
ii. There are sufficient securities to diversify away idiosyncratic risk, and
iii. Efficient and well-functioning security markets do not allow for persisting arbitrage opportunities.

3.2 The APT Model Formulation

The Arbitrage Pricing Theory postulates that there exists a linear relationship between the expected return of a security and the various macro-economic factors. The model holds the fact that the security returns are affected by a variety of risk factors instead of a single factor (index) of the market as in the case of CAPM and the Single Index Model. However, the APT model does not spell out these factors, but it is assumed that the relationship between security returns and the factor is linear.

According to the APT model, return on security i, is given by the following equation:

$$r_i = E(r_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \ldots\ldots\ldots.. + \beta_{in}F_n + e_i$$

.....Equation.1

Where, r_i = the rate of return on security i

$E(r_i)$ = expected rate of return on security i

F_k , k =1 to n , is the macroeconomic factor

β_{ik} = the sensitivity of the i^{th} security to the k^{th} factor

e_i = the unsystematic return on security i

For the above equilibrium to hold true, following conditions must be satisfied,

$E(e_i, e_n) = 0$ for all i and n where $i \neq n$

$E(e_i, F_k) = 0$ for all stocks and factors.

Ross derived the following relationship which is called the APT model,

$$E(r_i) = r_f + \beta_{i,1}RP_1 + \beta_{i,2}RP_2 + \dots\dots\dots + \beta_{i,n}RP_n$$

..... *Equation.2*

Where, $RP_k = [E(r_{Fk}) - r_f]$ is the risk premium for the k[th] systematic factor i.e. the excess of return on the k[th] systematic factor over the risk free rate.

The model asserts that the investors want to be compensated for all the risk factors that systematically affect the return of a security. The compensation is the aggregate of the product of the sensitivity of security to each risk factor ($\beta_{i,Fk}$) and the factor risk premium$[E(r_{Fk}) - r_f]$.

The rate of return thus derived from the model will then be employed for pricing the assets correctly. The asset price shall be equal to the asset price at the end of the expected period discounted at the rate derived by the model. Any divergence in the price of the asset shall be brought back to line through the process of arbitrage.

4. Arbitrage Principle

The Arbitrage Pricing theory is based on the principles of arbitrage. Arbitrage is the practice of earning riskless profit by taking advantage of price difference between two or more markets, without making any net investment. In other words, the arbitrage is the process of simultaneously buying and selling of an asset in two different markets having different prices. The arbitrageur makes a sure sort profit with no risk by buying in the market having low price and selling in a market having high price. It also implies that if due to certain imperfections, the securities with same risk level exhibit different returns, the investor would sell the security with low return and buy the security with high return.

A combined effect of the investors' utility maximization theorem, homogeneous beliefs of investor and perfect markets is that the securities are priced efficiently in the long run. The arbitrage opportunities exists in a situation where the payoffs (or expected return) from an asset is identical to an asset that is priced differently. The arbitrage relies on the fundamental principle of finance i.e. the "***law of one price***"***.*** The law states that two identical securities with same level of risk (σ_i) must have the same price i.e. mean return for each of the security must be same. However, the two identical securities might be priced differently because of the short disequilibrium and imperfections prevailing in the market, but this are possible only in the short run. This price difference in securities shall open the way to arbitrage opportunities in the market. Such an opportunity would be grabbed by the arbitrageurs and they shall engage in arbitrage activities. They shall simultaneously buy the securities where it is cheap and sell where it is expensive. Bidding up the asset price where it is low and forcing down the asset price where it is high will be continued until the arbitrageurs opportunities are eliminated completely. Thus, the securities shall be priced efficiently in the long run. The notion that the market prices will move to maintain no arbitrage opportunities is perhaps the most fundamental concept in the capital market theory.

. 5. Arbitrage Mechanics in APT

Arbitrage mechanism in APT model involves simultaneous trading of two assets one of which is mispriced. The mispriced asset is the one whose current price is different from the price predicted by the model. An asset is correctly priced if the price is equal to the sum of all future cash flows discounted at the rate of return specified by the APT model. The expected return derived in the APT model is linear function of various factors, and sensitivity to changes in each factor is depicted through factor specific beta coefficient. The arbitrageurs engage in trading between the correctly priced asset and mispriced asset by selling the asset that is relatively too expensive and buying the asset that is relatively too cheap and thus making a risk free profit which is equal to price difference between assets.

A correctly priced asset here refers to a synthetic asset- a *portfolio* consisting of other correctly priced assets. Such a portfolio is exposed to the macro economic factors in the same manner and extent as the mispriced asset i.e. the synthetic asset has a beta per factor equivalent to that of the mispriced asset.

The investor by simultaneously holding long position in asset and short position in portfolio (synthetic asset) and vice versa, earns a positive expected return that has net- zero exposure to macro-economic factors. The expected return thus earned is difference between the asset return and portfolio return. The arbitrageur thus makes a riskless profit by taking advantage of price difference between mispriced asset and correctly priced asset.

Where today's price is too low:
The implication is that at the end of the period the *portfolio* would have appreciated at the rate implied by APT, whereas the mispriced asset would have appreciated at *more* than this rate. The arbitrageur could therefore:

Today:

i. Short sell the portfolio
ii. Buy the mispriced asset with the proceeds

At the end of the period:

i. Sell the mispriced asset
ii. Use the proceeds to buy back the *portfolio*
iii. Pocket the difference

Where today's price is too high:
The implication is that at the end of the period the *portfolio* would have appreciated at the rate implied by APT, whereas the mispriced asset would have appreciated at *less* than this rate. The arbitrageur could therefore:

Today:

i. Short sell the mispriced asset
ii. Buy the *portfolio* with the proceeds

At the end of the period:

i. Sell the *portfolio*

ii. Use the proceeds to buy back the mispriced asset
iii. Pocket the difference

6. Arbitrage Portfolios

The arbitrage theory postulates that the investor attempts to increase the return from his portfolio without increasing the funds in the portfolio. At the same he wishes to keep the risk at the same level. Suppose an investor holds a portfolio of securities A, B and C and wants to change the proportion of securities without making any change in the financial commitment. The changes in the proportion of securities can be denoted by X_A, X_B and X_C. An increase in the proportion of one security is possible only when there is a decrease in the proportion of other security. Since the financial commitment is not increased, an increase in the proportion of investment in security A is possible only if he reduces the proportion of investment either in security B or C. The changes in the proportion of investment in different securities will add up to zero which is basic requirement of arbitrage portfolio. If X indicates the change in the proportion:

$$\Delta X_A + \Delta X_B + \Delta X_C = 0$$

..... Equation.3

The factor sensitivity in arbitrage model represents the responsiveness of a securities return to a particular portfolio. The sensitivities of securities to any factor are the weighted average of the sensitivities of securities. The weight of the sensitivity is the change in the proportion of securities. In an arbitrage portfolio the sum of sensitivities becomes zero.

$$b_A \Delta X_A + b_B \Delta X_B + b_C \Delta X_C = 0$$

..... Equation.4

Where, b_A, b_B, b_C are the sensitivities and ΔX_A, ΔX_B, ΔX_C are the changes in the proportion of securities.
Also the expected return in an arbitrage portfolio should be greater than zero.

$$\Delta X_A R_A + \Delta X_B R_B + \Delta X_C R_C > 0$$

.....Equation.5

Example 38.1: The investor holds A, B and C stocks with the following returns and the sensitivity to changes in the industrial production. The total amount invested is Rs. 1, 50,000.

Table 38.1:

I	R	B	Original Weights
Stock A	20%	0.45	.33
Stock B	15%	1.35	.33
Stock C	12%	.55	.34

Now the proportions are changed.
The changes are, ΔX_A= .2, ΔX_B =0.025, ΔX_C= -0.225.
For an arbitrage portfolio, $\Delta X_A + \Delta X_B + \Delta X_C = 0$

$$0.2 + 0.025 + (-0.225) = 0$$

The sensitivities also become zero,

$$b_A \, \Delta X_A + b_B \, \Delta X_B + b_C \, \Delta X_C = 0$$

$$0.45 * 0.2 + 1.35 * 0.025 + 0.55 * (-0.225) = 0$$

In an arbitrage portfolio the expected return should be greater than zero.

$$\Delta X_A R_A + \Delta X_B R_B + \Delta X_C R_C > 0$$

0.2* 20 + 0.025*15 + (-0.225)*12 > 0
4.375 – 2.7 > 0
i.e. 1.675%

The investor would increase his investment in stock A and B by selling C. The new composition of weights is, X_A = 0.53, X_B = 0.355, X_C = 0.115

7. The APT and the CAPM

Both the APT and the CAPM are prominent theories on asset pricing. Both theories give promising results in that they provide a benchmark rate of return that could be useful in capital budgeting, security evaluation, investment performance evaluation. In fact, APT has several advantages over the CAPM. It is less restrictive in its assumptions as compared to the CAPM. The CAPM is a single factor model and is derived by assuming an inherently observable market portfolio; that is difficult to construct. In contrast, APT model argues that one single factor cannot correctly influence the risk and return of security/portfolio. Rather, the security risk -return is influenced by a set of multiple factors. APT is thus called a multifactor model. The CAPM assumes that investors utilize a mean-variance approach. On the other hand, APT assumes that the asset prices can be influenced by factors beyond means and variances. In a way, CAPM may be called a special case of APT, in that the security market line of CAPM reflects a single factor model, where β is the sensitivity to market movement.
In spite of the several advantages, the APT does not fully overcome the CAPM. Since APT focuses on the no arbitrage condition, without the assumptions of the market or index model, the expected return beta relationship for any particular asset cannot be ruled out. The CAPM and APT are not really at odds to each other. The two models are in fact complementary to each other rather than competing. Both the models assert positive expected return result from the factor sensitivities to the market movements and vice-versa.

8. Summary

- The APT model postulates that security returns are influenced by multiple macroeconomic factors where sensitivities to each factor is represented by factor specific beta coefficient.
- The APT model holds that there exists a linear relationship between the return on security and the number of risk factors.
- The expected rate of return thus derived from the APT model will then be employed for pricing the assets correctly.
- The model emphasizes the law of one price and no arbitrage condition in the long run.
- Arbitrage opportunity occurs when there are differences in asset prices in two or more markets.
- Arbitrageurs shall bid up the asset price where it is low (long position) and force down the asset price where it is high (short selling).
- The arbitrage portfolio is constructed without any increase in the financial commitment.
- For an arbitrage portfolio,

$$\Delta X_A + \Delta X_B + \Delta X_C = 0$$

$$b_A \Delta X_A + b_B \Delta X_B + b_C \Delta X_C = 0$$

$$\Delta X_A R_A + \Delta X_B R_B + \Delta X_C R_C > 0$$

- The APT does not require the restrictive assumptions of the CAPM and its market portfolio.